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# **Intuitionistic Q-Fuzzy Normal Subgroups**

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**ABSTRACT:** In this paper, we introduce the concept of intuitionistic Q-fuzzy normal group and discussed some of its properties

**KEYWORDS**: Fuzzy set, , fuzzy subgroup, ,Q-fuzzy subgroup , intuitionistic Q- fuzzy subgroups , Q-fuzzy normal subgroup,Q-fuzzy normalizer , intuitionistic Q- fuzzy normal subgroups, intuitionistic Q- fuzzy left and right cosets

### I. INTRODUCTION

After the introduction of the concept of fuzzy sets by L.A.Zadeh [6], researchers were conducted the generalizations of the notion of fuzzy sets, A. Rosenfeld [7] introduced the concept of fuzzy group and the idea of "intuitionistic fuzzy set" was first published by K.T. Atanassov [5]. S..Also T. Priya, T.Ramachandran, K.T. Nagalakshmi" *On Q-Fuzzy Normal Subgroups*" [3]..A.Solairaju and R. Nagarajan [1,2] introduce and define a new algebraic structure of Q-fuzzy normal subgroups and Q-fuzzy left and right cosets. Choudhury.F.P. and Chakraborty.A.B. and Khare.S.S.,[4] *A note on fuzzy subgroups and Homomorphism Journal of mathematical analysis and applications* In this chapter we introduce the concept of intuitionistic Q-fuzzy normal subgroups and some properties are proved

#### **II. PRELIMINARIES**

### 2.1. Definition:Fuzzy Set

Let X be a non-empty set. A fuzzy set  $\mu$  on X is a mapping  $\mu: X \to [0,1]$  and is defined as

$$\mu = \left\{ x \in X / (x, \mu(x)) \right\}$$

### 2.2. Definition:Fuzzy Level Subset

Let  $\mu$  be a fuzzy subset of a set X. For  $t \in [0,1]$ ,  $\mu_t = \{x \in X \mid \mu(x) \ge t\}$  is called a **level fuzzy subset** of  $\mu$ 

### 2.3. Definition: Fuzzy Multiset (FMS)

Let X be a non-empty set. A Fuzzy Multiset (FMS) A drawn from X is characterized by a function 'Count membership' of A denoted by  $CM_A$  such that  $CM_A: X \to Q$  where Q is the set of all crisp finiteset drawn from the unit interval [0,1]. Then for any  $x \in X$ , the value  $CM_A(x)$  is a crisp multiset drawn from



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[0,1]. For each  $x \in X$ , the membership sequence is defined as the decreasingly ordered sequence of elements

in 
$$CM_A(x)$$
. It is denoted by  $\left(\mu_{A_1}(x), \mu_{A_2}(x), \dots, \mu_{A_k}(x)\right)$  where  
Example 2.3

### Example:2.3

Let  $X = \{x, y, z, w\}$  be a universal non empty set. For each  $x \in X$ , we can write a Fuzzy Multi set as follows  $A = \{ \langle x, (0.8, 0.7, 0.7, 0.6) \rangle, \langle y, (0.8, 0.5, 0.2) \rangle, \langle z, (1, 0.5, 0.5) \rangle \}$  Where

 $CM_A(x) = (0.8, 0.7, 0.7, 0.6)$  with  $0.8 \ge 0.7 \ge 0.7 \ge 0.6$ 

### 2.4. Definition: Q-Fuzzy set

A mapping  $\mu: M \times Q \rightarrow [0,1]$ , where M and Q are arbitrary non-empty sets is called

**Q-fuzzy set** of *M* and is denoted by  $A_Q = \{ [(x,q), \mu(x,q)] | x \in M, q \in Q \}$ 

### Example:2.4

Let  $M = \{m_1, m_2, m_3, m_4, m_5\}$  and  $Q = \{p, q, r\}$  be any two arbitrary non-empty sets, then  $A_o = \{ \lceil (m_1, p), 0.3 \rceil, \lceil (m_1, q), 0.6 \rceil, \lceil (m_1, r), 0.4 \rceil, \lceil (m_2, p), 0.3 \rceil, \rceil \}$  $[(m_2,q),0.3],[(m_2,r),0.6],[(m_3,r),0.4],[(m_4,p),0.3]\}$ 

### 2.5. Definition: Intuitionistic Fuzzy set (IFS)

Let X be a non empty set. An Intuitionistic Fuzzy set A on X is an object having the form  $A = \left\{ \left\langle x, \mu_A(x), \gamma_A(x) \right\rangle / x \in X \right\}, \quad \text{where} \quad \mu_A : X \to [0,1] \And \gamma_A : X \to [0,1] \quad \text{are} \quad \text{the} \quad \text{degree of} \quad X \to [0,1] \land Y \to [0,1] \quad \text{are} \quad \text{the} \quad \text{degree of} \quad Y \to [0,1] \land Y \to [0,1] \quad \text{are} \quad Y \to [0,1] \quad \text{are} \quad Y \to [0,1] \quad \text{the} \quad Y \to [0,1] \quad Y \to [0,1] \quad \text{the} \quad Y \to [0,1] \quad \text{the} \quad Y \to [0,1] \quad$ membership and non-membership functions respectively with  $0 \le \mu_A(x) + \gamma_A(x) \le 1$ 

### 2.6. Definition: Intuitionistic Q- Fuzzy set (IQFS)

Let X and Q are arbitrary non-empty sets. An Intuitionistic Q-Fuzzy Set (IQFS in short) A is an object having  $A = \left\{ \left\langle (x,q), \mu_A(x,q), \gamma_A(x,q) \right\rangle : x \in X, q \in Q \right\}$ the form where the functions  $\mu_A: X \times Q \to [0,1]$  and  $\gamma_A: X \times Q \to [0,1]$  denote the degree of membership and non-membership of each  $(x,q) \in X \times Q$  to the element set Α respectively, and for all  $x \in X$  and  $q \in Q$ ,  $0 \le \mu_A(x,q) + \gamma_A(x,q) \le 1$ 

### 2.9. Definition:

Let G be a group. A fuzzy subset A of G is said to be a fuzzy subgroup of G if

$$(i).A(xy) \ge \min\left\{A(x), A(y)\right\}$$

(*ii*).
$$A(x^{-1})^3 A(x) = x, y \hat{\mathbf{I}} G$$

### 2.7. Definition:

Let G be a group. A fuzzy subset  $\mu$  of G is said to be a **normal fuzzy subgroup** of G if

(*i*).
$$\mu(xyx^{-1}) = \mu(y)$$
 or  $\mu(xy) = \mu(yx)$ 



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#### 2.8. Definition:

A Q-fuzzy set  $\mu$  is called a Q-fuzzy subgroup of a group G if

$$(i).\,\mu(xy,q) \ge \min\left\{\mu(x,q),\,\mu(y,q)\right\}$$

(*ii*). $m(x^{-1},q)^3 m(x,q)$  "  $x, y \hat{\mathbf{I}} G, q \hat{\mathbf{I}} Q$ 

#### 2.9. Definition:

Let  $\mu$  be a Q-fuzzy subgroup of a group G. For  $t \in [0, 1]$ , we define the level subset of  $\mu$  is the set  $\mu^t = \{x \in G, q \in Q/\mu(x, q) \ge t\}$ 

#### 2.10. Definition:

Let  $\mu$  be a multi Q-fuzzy subgroup of a group G. For  $t \in [0, 1]$ , we define the multi level subset of  $\mu$  is the set  $\mu_i^t = \left\{ x \in G, q \in Q / \mu_i(x, q) \ge t \right\}$ 

### 2.11. Definition: Intuitionistic Q-Fuzzy Level subsets

Let  $A = \left\{ \left\langle \left(x,q\right), \mu_A(x,q), \gamma_A(x,q) \right\rangle : x \in X, q \in Q \right\}$  be a IQFS on X. Then for  $\alpha, \beta \in [0,1]$ , the set  $A^{[\alpha,\beta]} = \left\{ x \in X, q \in Q \mid \mu_A(x,q) \ge \alpha \text{ and } \gamma_A(x,q) \le \beta \right\}$  is called the  $(\alpha,\beta)$ -level subsets of A.

#### Theorem:3.1

Let G be a group and A be a Intuitionistic Q-fuzzy subset of G. Then A is a Intuitionistic Q-fuzzy subgroup of a group G iff the Intuitionistic  $(\alpha, \beta)$ -level subsets  $A^{[\alpha, \beta]}$   $\alpha, \beta \in [0, 1]$ , are subgroups of G

#### **Proof:**

Let A be a Intuitionistic Q- fuzzy subgroup of G and the  $(\alpha, \beta)$ - level subset  $A^{[\alpha,\beta]} = \{x \in X, q \in Q / \mu_A(x,q) \ge \alpha \text{ and } \gamma_A(x,q) \le \beta\}.$ Let  $x, y \in A^{[\alpha,\beta]}$ . Then  $\mu_A(x,q) \ge \alpha$  and  $\gamma_A(x,q) \le \beta$ Now,  $\mu_A(xy^{-1},q) \ge \min \{\mu_A(x,q), \mu_A(y^{-1},q)\}$   $= \min \{\mu_A(x,q), \mu_A(y,q)\}$   $= \min \{\alpha, \alpha\} = \alpha$ Hence  $\mu_A(xy^{-1},q) \ge \alpha$ . Similarly,  $\gamma_A(xy^{-1},q) \le \max \{\gamma_A(x,q), \gamma_A(y^{-1},q)\}$   $= \max \{\gamma_A(x,q), \gamma_A(y,q)\}$  $= \max \{\beta, \beta\} = \beta$ 

Hence  $\gamma_A(xy^{-1},q) \leq \beta$ .



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This implies 
$$xy^{-1} \in A^{[\alpha,\beta]}$$
.  
Thus  $A^{[\alpha,\beta]}$  is a subgroup of  $G$ .  
Conversely, let us assume that  $A^{[\alpha,\beta]}$  is a subgroup of  $G$   
Let  $x, y \in A^{[\alpha,\beta]}$ . Then  $\mu_A(x,q) \ge \alpha$  and  $\gamma_A(x,q) \le \beta$   
Also,  $\mu_A(xy^{-1},q) \ge \alpha = \min \{\alpha, \alpha\}$   
 $= \min \{\mu_A(x,q), \mu_A(y,q)\}$   
 $\mu_A(xy^{-1},q) \ge \min \{\mu_A(x,q), \mu_A(y,q)\}$   
Also,  $\gamma_A(xy^{-1},q) \le \beta = \max \{\beta, \beta\}$   
 $= \max \{\gamma_A(x,q), \gamma_A(y,q)\}$   
 $\gamma_A(xy^{-1},q) \le \max \{\gamma_A(x,q), \gamma_A(y,q)\}$   
Hence  $A$  is a Intuitionistic  $Q$  – fuzzy subgroup of  $G$ 

2.12. Definition:

Let G be a group and A be a Intuitionistic Q-fuzzy subgroup of a group G.Let  $N(A) = \left\{ a \in G / \mu_A(axa^{-1}, q) = \mu_A(x, q), \gamma_A(axa^{-1}, q) = \gamma_A(x, q) \, \forall x \in G, q \in Q \right\}.$ Then N(A) is called the Intuitionistic Q-fuzzy normalizer of A

### Theorem: 3.2

Let G be a group and A be a Intuitionistic Q-fuzzy subset of G. Then A is a Intuitionistic Q-fuzzy normal subgroup of G iff the level subsets  $A^{[\alpha,\beta]}$ ,  $\alpha, \beta \in [0,1]$  is a subgroup of G **Proof:** 

Let *A* be a Intuitionistic *Q*-fuzzy normal subgroup of *G* and the level subsets  $A^{[\alpha,\beta]}$ ,  $\alpha, \beta \in [0,1]$ , is a subgroup of *G*. Let  $x \in G$  and  $a \in A^{[\alpha,\beta]}$ , then  $\mu_A(a,q) \ge \alpha$ . Now  $\mu_A(xax^{-1},q) = \mu_A(a,q) \ge \alpha$ Since *A* a Intuitionistic *Q*-fuzzy normal subgroup of *G* That is,  $\mu_A(xax^{-1},q) \ge \alpha$  $\Rightarrow xax^{-1} \in A^{[\alpha,\beta]}$ 

Similarly,  $\gamma_A(xax^{-1},q) = \gamma_A(a,q) \le \beta$ 

Since A a Intuitionistic Q-fuzzy normal subgroup of G

That is,  $\gamma_A(xax^{-1}, q) \leq \beta$ 

 $\Rightarrow xax^{-1} \in A^{[\alpha,\beta]}$ 

Hence  $A^{[\alpha,\beta]}$  is a normal subgroup of G

### Theorem:3.3

Let A be a Intuitionistic Q-fuzzy normal subgroup of a group G. Then

(i). N(A) is a subgroup of a group G.

(ii). A is a Intuitionistic Q-fuzzy normal subgroup of a group G iff N(A) = G.



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(iii). A is a Intuitionistic Q-fuzzy normal subgroup of a group N(A). **Proof:** Let  $a, b \in N(A) \Rightarrow \mu_A(xax^{-1}, q) \ge \alpha$ ,  $\mu_A(xbx^{-1}, q) \ge \alpha$ , for all  $x \in G, q \in Q$ . Now,  $\mu_{A}(abx(ab)^{-1}, q) = \mu_{A}(abxb^{-1}a^{-1}, q) \Longrightarrow$  $= \mu_{A}(bxb^{-1}, q) = \mu_{A}(x, q)$ Thus  $\mu_{A}(abx(ab)^{-1}, q) = \mu_{A}(x, q)$ Let  $a, b \in N(A) \Rightarrow \gamma_A(xax^{-1}, q) \le \beta$ ,  $\gamma_A(xbx^{-1}, q) \le \beta$ , for all  $x \in G, q \in Q$ . Now,  $\gamma_{A}(abx(ab)^{-1}, q) = \gamma_{A}(abxb^{-1}a^{-1}, q) \Longrightarrow$  $= \gamma_{A}(bxb^{-1}, q) = \gamma_{A}(x, q).$ Thus  $\gamma_A(abx(ab)^{-1}, q) = \gamma_A(x, q)$ This implies  $ab \in N(\lambda)$ That is N(A) is a subgroup of a group G (ii). From (i).  $N(A) \subseteq G$ , A is a Intuitionistic Q-fuzzy normal subgroup of a group G. Let  $a \in G \Longrightarrow \mu_A(axa^{-1}, q) = \mu_A(x, q) \Longrightarrow a \in N(A)$ .  $\gamma_A(axa^{-1},q) = \gamma_A(x,q) \Longrightarrow a \in N(A)$ Hence N(A) = G. Conversely let N(A) = G. Clearly  $\mu_A(axa^{-1},q) = \mu_A(x,q)$ , and  $\gamma_A(axa^{-1},q) = \gamma_A(x,q) \quad \forall a, x \in G, q \in Q$ Hence A is a Intuitionistic Q – fuzzy normal subgroup of group G. From (2), A is a Intuitionistic Q-fuzzy normal subgroup of a group N(A)Theorem: 3.4 If A is a Intuitionistic Q-fuzzy subgroup of a group G. Then  $g\mu_A g^{-1}$  &  $g\gamma_A g^{-1}$  are also a Intuitionistic Qfuzzy subgroups of a group G for all  $g \in G$  and  $q \in Q$ . **Proof:** Let A is an Intuitionistic Q – fuzzy subgroup of a group G Then (i).  $(g\mu_{A}g^{-1})(xy, q) = \mu_{A}(g^{-1}(xy)g, q)$  $=\mu_{A}(g^{-1}(xgg^{-1}y)g, q)$  $=\mu_{A}((g^{-1}xg)(g^{-1}yg), q)$  $\geq \operatorname{Min}\left\{\mu_{A}\left(g^{-1}xg,q\right), \ \mu_{A}\left(g^{-1}yg,q\right)\right\}$ for all  $\forall x, y \in G$  and  $q \in Q$ Similarly,  $(g\gamma_A g^{-1})(xy, q) = \gamma_A(g^{-1}(xy)g, q)$  $=\gamma_{A}(g^{-1}(xgg^{-1}y)g, q)$ 

$$= \gamma_{A} \left( (g^{-1}xg)(g^{-1}yg), q \right)$$
  

$$\leq \operatorname{Max} \left\{ \gamma_{A} \left( g^{-1}xg, q \right), \gamma_{A} \left( g^{-1}yg, q \right) \right\}$$



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for all 
$$\forall x, y \in G$$
 and  $q \in Q$   
(ii).  $(g \mu_A g^{-1})(x, q) = \mu_A (g^{-1} x g, q)$   
 $= \mu_A ((g^{-1} x g)^{-1}, q)$   
 $= \mu_A (g^{-1} x^{-1} g, q)$   
 $= g \mu_A g^{-1} (x^{-1}, q)$   
Similarly,  $(g \gamma_A g^{-1})(x, q) = \gamma_A (g^{-1} x g, q)$   
 $= \gamma_A ((g^{-1} x g)^{-1}, q)$   
 $= \gamma_A (g^{-1} x^{-1} g, q)$   
 $= g \gamma_A g^{-1} (x^{-1}, q)$ 

for all  $\forall x, y \in G$  and  $q \in Q$ .

Hence  $g \mu_A g^{-1} \& g \gamma_A g^{-1}$  are also a Intuitionistic Q – fuzzy subgroups of a group G. **Theorem:3.5:** 

Let A and B be two Intuitionistic Q-fuzzy subgroup of a group G. Then  $A \cap B$  is also an Intuitionistic Q-fuzzy subgroup of a group G

**Proof:** 

Let A and B be two Intuitionistic Q – fuzzy subgroups of a group G

$$\begin{aligned} \text{(i).} (A \cap B)(xy^{-1}, q) &= \operatorname{Min} \left\{ \mu_{A}(xy^{-1}, q), \mu_{B}(xy^{-1}, q) \right\} \\ &\geq \operatorname{Min} \left\{ \min \{ \mu_{A}(x, q), \mu_{A}(y, q) \}, \min \{ \mu_{B}(x, q), \mu_{B}(y, q) \} \right\} \\ &\geq \operatorname{Min} \left\{ \min \{ \mu_{A}(x, q), \mu_{B}(x, q) \}, \min \{ \mu_{A}(y, q), \mu_{B}(y, q) \} \right\} \\ &= \operatorname{Min} \left\{ (A \cap B)(x, q), (A \cap B)(y, q) \right\} \\ &= \operatorname{Min} \left\{ (A \cap B)(xy^{-1}, q) \geq \operatorname{Min} \left\{ (A \cap B)(x, q), (A \cap B)(y, q) \right\} \\ &= \operatorname{Max} \left\{ nax \{ \gamma_{A}(x, q), \gamma_{A}(y, q) \}, max \{ \gamma_{A}(xy^{-1}, q), \gamma_{B}(xy^{-1}, q) \} \right\} \\ &\leq \operatorname{Max} \left\{ max \{ \gamma_{A}(x, q), \gamma_{A}(y, q) \}, max \{ \gamma_{B}(x, q), \gamma_{B}(y, q) \} \right\} \\ &= \operatorname{Max} \left\{ (A \cap B)(x, q), (A \cap B)(y, q) \right\} \\ &= \operatorname{Max} \left\{ (A \cap B)(x, q), (A \cap B)(y, q) \right\} \\ &= \operatorname{Min} \left\{ (A \cap B)(x, q) = \operatorname{Min} \{ \mu_{A}(x, q), \mu_{B}(x, q) \} = \operatorname{Min} \{ \mu_{A}(x^{-1}, q), \mu_{B}(x^{-1}, q) \} \\ &= \operatorname{Min} \left\{ (A \cap B)(x, q) = \operatorname{Max} \{ \gamma_{A}(x, q), \gamma_{B}(x, q) \} = \operatorname{Max} \{ \gamma_{A}(x^{-1}, q), \gamma_{B}(x^{-1}, q) \} \\ &= \operatorname{Max} \left\{ (A \cap B)(x^{-1}, q) \right\}. \end{aligned}$$

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Hence  $A \cap B$  is also an Intuitionistic Q – fuzzy subgroup of a group G

### Theorem: 3.6

The intersection of any two Intuitionistic Q-fuzzy normal subgroup of a group G is also a Intuitionistic Q-fuzzy normal subgroup of a group G.

### **Proof:**

Let A and B be two Intuitionistic Q – fuzzy normal subgroup of a group G

According to previuos theorem,  $A \cap B$  is also a Intuitionistic Q – fuzzy subgroup of a group G

Now we have to prove that  $A \cap B$  is a normal subgroup

Now  $\forall x, y \in G$  and  $q \in Q$  we have

$$(A \cap B)(xyx^{-1}, q) = \min \{\mu_A(xyx^{-1}, q), \mu_B(xyx^{-1}, q)\}$$

$$=\min\{\mu_{A}(y,q),\mu_{B}(y,q)\}$$

$$=(A \cap B)(y,q)$$

Similarly,  $\forall x, y \in G$  and  $q \in Q$  we have

$$(A \cap B)(xyx^{-1}, q) = \operatorname{Max}\left\{\gamma_{A}(xyx^{-1}, q), \gamma_{B}(xyx^{-1}, q)\right\}$$

 $= \operatorname{Max}\left\{\gamma_{A}(y, q), \gamma_{B}(y, q)\right\}$ 

$$=(A \cap B)(y,q)$$

Hence  $A \cap B$  is a Intuitionistic Q – fuzzy normal subgroup of a group G

### 3.7. Definition:

The mapping  $f: G \times Q \to H \times Q$  is said to be a group Intuitionistic Q-fuzzy homomorphism if

(i).  $f: G \to H$  is a group homomorphism

(ii). 
$$f(xy, q) = (f(x)f(y), q) \quad \forall x, y \in G, q \in Q$$

### Theorem: 3.7

Let  $f: G \times Q \to H \times Q$  be Intuitionistic Q-fuzzy group homomorphism

(i). If A is a Intuitionistic Q-fuzzy normal subgroup of a group H, then f(A) is a Intuitionistic Q-fuzzy normal subgroup of a group G,

(ii). If f is an epimorphism and A is a Intuitionistic Q-fuzzy normal subgroup of a roup G, then f(A) is a Intuitionistic Q-fuzzy normal subgroup of a group H

### Proof:

Let  $f: G \times Q \to H \times Q$  be a group Intuitionistic Q-fuzzy homomorphism and let A be a Intuitionistic Q-fuzzy normal subgroup of a group H.

Now, for all 
$$x, y \in G$$
,  $q \in Q$  we have  $f'(\mu_A)(xyx^{-1}, q) = \mu_A(f(xyx^{-1}, q))$   
 $= \mu_A(f(x)f(y)(f(x))^{-1}, q)$   
 $= \mu_A(f(y), q)$ 



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$$= f'(\mu_A)(y, q)$$
  
Similarly,  $f'(\gamma_A)(xyx^{-1}, q) = \gamma_A(f(xyx^{-1}, q))$ 
$$= \gamma_A(f(x)f(y)(f(x))^{-1}, q)$$
$$= \mu_A(f(y), q)$$
$$= f'(\gamma_A)(y, q)$$

Hence f'(A) is a Intuitionistic Q-fuzzy normal subgroup of a group G ,

(ii).Let A be a Intuitionistic Q-fuzzy normal subgroup of a group G, then f(A) is a Intuitionistic Q-fuzzy subgroup of a group H

Now, for all  $u, v \in H$  we have

$$f(\mu_{A})(uvu^{-1}, q) = \sup_{f(\mu_{A})=uvu^{-1}} \mu_{A}(y,q) = \sup_{f(x)=u;f(y)=v} \mu_{A}(y,q)$$
$$= \sup_{f(y)=v} \mu_{A}(y,q) = f(\mu_{A})(v,q) \text{ since } f \text{ is an epimorphism}$$
$$f(\gamma_{A})(uvu^{-1}, q) = \inf_{f(\gamma_{A})=uvu^{-1}} \gamma_{A}(y,q) = \inf_{f(x)=u;f(y)=v} \gamma_{A}(y,q)$$
$$= \inf_{f(y)=v} \gamma_{A}(y,q) = f(\gamma_{A})(v,q) \text{ since } f \text{ is an epimorphism}$$

Hence  $f(\mu_A, \gamma_A)$  is a Intuitionistic Q – fuzzy normal subgroup of a group H.

#### **III. CONCLUSION**

In this article we have discussed Intuitionistic Q-fuzzy normal subgroups, n-generated Q-fuzzy Normalizer and Intuitionistic Q-fuzzy subgroups under homomorphism. Interestingly, it has been observed that Q-fuzzy concept adds an another dimension to the defined fuzzy normal subgroups. This concept can further be extended for new results

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